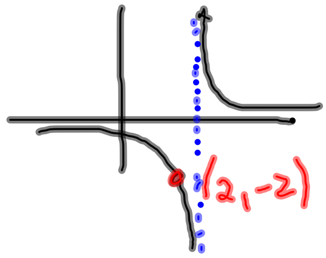
8.2 l'hôpital's rule

ex.

$$\lim_{x\to 2} \frac{\sin(x-2)}{(x-2)^2 - \frac{1}{2}(x-2)} = ?$$



$$h(x) = \frac{\sin(x-2)}{(x-2)^2 - \frac{1}{2}(x-2)}$$

$h(2) = *$, $\lim_{x\to 2} h(x) = -2$

hole

l'hopital's

$$\lim_{x\to 2} \frac{\cos(x-2)}{2(x-2) - \frac{1}{2}} = \frac{1}{-\frac{1}{2}} = -2$$

why did it work?

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replace f(x), g(x) with their tan lines

$$\lim_{x\to 2} \frac{f(x)}{g(x)} = \lim_{x\to 2} \frac{f'(2)\cancel{(x-2)} + \overset{0}{f(2)}}{g'(2)\cancel{(x-2)} + \underset{0}{g(2)}}$$

l'hopital's rule (first form)

$\boxed{\begin{array}{l}\text{if } f(a) = g(a) = 0, \text{ and } f'(x) \text{ \& } g'(x) \text{ exist and } g'(a) \neq 0 \\ \text{then } \lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}\end{array}}$

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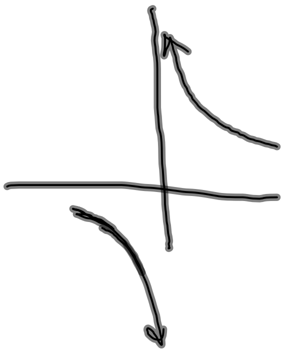
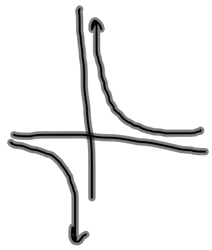
$$\lim_{x \to 0} \frac{\sin x}{x^2} = \infty \text{ — undefined}$$

$$\lim_{x \to 0^{\pm}} \frac{\cos x \to 1}{2x \to 0} = \pm\infty$$



$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$



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stronger version $\displaystyle \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Ex 4

$\frac{0}{0}$, $\frac{\infty}{\infty}$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec x \to \infty}{1 + \tan x \to \infty} = \lim_{x \to \frac{\pi}{2}} \frac{\cancel{\sec x} \tan x}{\sec^{\cancel{2}} x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \lim_{x \to \frac{\pi}{2}} \sin x = 1$$

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other indet. $\infty \cdot 0$, $\infty - \infty$, 1^∞ , 0^0 , ∞^0

1^∞

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = y$$

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x = \ln y$$

$$\lim_{x\to\infty} x \cdot \ln\left(1+\frac{1}{x}\right) = \ln y$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \ln y$$

0

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \ln y$$

0

$$1 = \ln y$$

$$e = y$$

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